Abstract

Aldous [(2007) Preprint] defined a gossip process in which space is a discrete $N \times N$ torus, and the state of the process at time t is the set of individuals who know the information. Information spreads from a site to its nearest neighbors at rate 1/4 each and at rate $N^{-\alpha}$ to a site chosen at random from the torus. We will be interested in the case in which $\alpha < 3$, where the long range transmission significantly accelerates the time at which everyone knows the information. We prove three results that precisely describe the spread of information in a slightly simplified model on the real torus. The time until everyone knows the information is asymptotically $T = (2 - 2\alpha/3)N^{\alpha/3}\log N$. If ρ_s is the fraction of the population who know the information at time s and ε is small then, for large N, the time until ρ_s reaches ε is $T(\varepsilon) \approx T + N^{\alpha/3}\log(3\varepsilon/M)$, where M is a random variable determined by the early spread of the information. The value of ρ_s at time $s = T(1/3) + tN^{\alpha/3}$ is almost a deterministic function $h(t)$ which satisfies an odd looking integro-differential equation. The last result confirms a heuristic calculation of Aldous.